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یک مدل دو هدفه MILP برای اندازه‌گیری و زمان‌بندی تولید: رویکرد برنامه‌ریزی آرمانی فازی احتمالی

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چکیده

در این مقاله یک مدل برنامه‌ریزی خطی عدد صحیح مختلط دو هدفه برای مسئله تعیین اندازه و زمان‌بندی تولید برای صنعت ماست به عنوان یک صنعت خوراکی فاسد شدنی تحت شرایط عدم اطمینان تقاضا ارائه می شود. اهداف مدل پیشنهادی، به حداقل رساندن همزمان هزینه کل و زمان اتمام تولید است. مدل پیشنهادی، بسیاری از ویژگی‌های متمایز فرآوری ماست را شامل می‌شود، از جمله ماندگاری ، تنظیمات ، نرخ بسته‌بندی ، حداقل و حداکثر اندازه تولید ، زمان آینده برای نگهداری محصولات و تقاضای فازی. علاوه بر این، مدل پیشنهادی ، شامل کنترل موجودی چند محصول و چند دوره است. از این رو، به عنوان یک مدل عملیاتی – استراتژیک طبقه‌بندی می‌شود. ما یک رویکرد ترکیبی متمرکز بر برنامه‌ریزی امکانی فازی و برنامه‌ریزی آرمانی فازی را برای حل مدل پیشنهادی دو هدفه ارائه می‌کنیم ، جایی که اقدامات امکان ، ضرورت و اعتبار مطابق با ترجیح تصمیم‌گیرندگان اتخاذ می‌شود. در مقایسه با مدل سنتی اندازه و زمان‌بندی تولید، تصمیم‌گیری و تجزیه‌وتحلیل حساسیت بهتری را می توان برای DMs بر اساس داده‌های سه مقدار کارایی به دست آمده، انجام داد. داده‌های به دست آمده از صنعت ماست، برای ارزیابی امکان‌سنجی مدل پیشنهادی و رویکرد راه حل استفاده شد. نتایج به دست آمده از اعمال روش و تجزیه‌وتحلیل حساسیت، اثربخشی مدل ریاضی و همچنین روش پیشنهادی را نشان داد.

کلید واژه‌ها: مسئله اندازه و زمان‌بندی تولید، عدم قطعیت، برنامه‌ریزی امکانی فازی، برنامه‌ریزی آرمانی

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A bi-objective MILP model for lot sizing and scheduling problem: Possibilistic fuzzy goal programming approach

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Abstract

This paper proposes a bi-objective mixed-integer linear programming model for formulating a lot- sizing and scheduling problem for the perishable yogurt industry under demand uncertainty. The objectives of the proposed model are to simultaneously minimize the overall cost and the total production completion time. The proposed MILP formulation integrates many distinctive features of yogurt processing, including shelf-life constraints, setups, packaging rates, minimum and maximum lot size limits, future time for holding products, and fuzzy demand. Additionally, the proposed model, including inventory control, is a multi-product and multi-period model hence, it is categorized as an operational-strategic model. We introduce a hybrid approach focused on fuzzy possibility programming and a fuzzy goal programming approach for solving the suggested bi-objective model, where possibility, necessity and credibility measures are adopted according to the decision makers' preference. Compared to the traditional model of lot sizing and scheduling, better decision-making and sensitivity analysis for DMs can be made based on the three obtained efficiency values. Data from the yogurt plant were used to assess the feasibility of the proposed model and solution approach. The results obtained from applying the method and sensitivity analysis showed the effectiveness of the mathematical formulation as well as the proposed solution method.

Keywords: Lot sizing and scheduling problem, Perishable products, Yogurt plant, Uncertainty, Fuzzy possibility programming, Goal programming approach

1. Introduction

The food industry is an important industrial activity and among the sustainable and fast-growing industries. Day-by-day newer innovations are being launched to get better quality of foods, especially dairy products including yogurt. The high market competition and the rapid growth of the product range, the profile of demand, and the perishability of raw material and finished goods make the food industry distinct from other industries. Poor production planning and production schedules have caused both material trash and supply insufficiency. As a consequence of these considerations, organizations are investigating on the possibilities of turning those complexities into strategic features through progressively enhancing their activity. The effective combination of production and scheduling planning is one of the key tasks for achieving this objective [1], [2].

In the food industry, where sequencing-dependent changeovers (setups) are very important, the decision of lot sizing and scheduling, which is, generally, taken heuristically, is a very hard task and often failed, where decision-makers solve the problem of lot-sizing sequentially first without considering the changeovers (setup times), and then the problem of scheduling. Since setups are not adequately planned for, so infeasible schedules can be produced. As a result, the decision-makers have to re-initiate the procedure, and even after several attempts, a near-optimal schedule is only reached [3], [4]. Use of an accurate mathematical approach that integrates all manufacturing criteria enables the incorporation of the problem of lot-sizing and scheduling, and thus, contributing to further feasible solutions. The use of simultaneous production planning and scheduling is, therefore, effective as proposed by a number of similar studies (e.g. [1], [7], [9], [12], and [25]).

When the perishability of products is taken into consideration, the issue of lot size and schedules becomes more complicated and challenging [6]. The perishability of products is of great importance in the food industry. Inventory management and inter-related strategic decisions are directly influenced by limited product shelf-life [31]. Lütke Entrup et al. [6] indicated that yogurt is a perishable product, and in order to calculate the market value of the final product, it creates a shelf-life-dependent pricing component. Amorim et al. [5], [21] investigated the perishability aspects related to diary and yogurt processing in the food sector.

On the other hand, uncertainty is addressed in this paper as a crucial issue. Demand uncertainty in the yogurt industry is typical and neglecting the uncertainty of demand can also be considered as a modeling error. This modeling error can lead to a rise in costs, and even lost sales and unsatisfied customers. The complication of dealing with the problem of lot sizing and scheduling with uncertainty is to make a decision on how to present uncertainty parameters among the different approaches. Uncertainty in demand could be designed by stochastic programming and fuzzy programming. There is currently little research on the subject of lot size and scheduling that considers the uncertainty of demand. Douglas Alem et al. [23] introduced the General Lot-Sizing and Scheduling Problem (GLSP) under demand uncertainty using a robust optimization budget-uncertainty set and a multistage stochastic programming. Via the Monte Carlo simulation process, the advantage of each technique was measured. Curcio et al. [24] studied a multi-stage stochastic programming model. The uncertainty of demand was represented using the moment-matching technique by scenario trees. Scenario reduction was used to select the scenarios that best represent the initial set.

In many cases, decision-makers do not have sufficient information for adjusting the demands' probability distribution, which restricts the effectiveness of stochastic programming in cooperation with uncertainty [22]. However, decision-makers prefer to get an unknown parameter estimated on their own, which can be pessimistic, optimistic, and moderately optimistic to predict. Using fuzzy logic to overcome uncertainty based on expert knowledge and insights is also advantageous [32, 34].

The outcome of this study is a model that uses a bi-objective mixed-integer linear programming the model is developed for formulating a lot-sizing and scheduling for the yogurt manufacturing under uncertain demand in order to minimize the overall cost of the production and minimize the completion time of the products that can also affect energy consumption. Generally, the main contributions of the present study are as follows:

• Designing a lot-sizing and scheduling model for multi-product, multi-period perishable yogurt industry considering demand under uncertainty.

• Proposing a fuzzy possibility programming and a fuzzy goal programming approach to solve the suggested bi-objective model under uncertainty.

• Conducting the suggested formulation and solution method in a case study and evaluating its effectiveness.

The remainder of the work is structured as follows. The literature relevant to the problem is discussed in part 2. Problem formulation is presented in Part 3. The solution method is provided in part 4. The suggested model is tested and validated in part 5. Part 6 is devoted to the results report. Part 7 discusses the sensitivity analysis. Finally, part 8 outlines the conclusions.

2. Literature review

2.1. Simultaneous lot-sizing and scheduling

Due to its relevance in the financial system and considering the complexities of solving it in many realistic ways, the simultaneous lot-sizing and scheduling issue has gained a lot of research interest, particularly with regard to sequence-dependent setups. Copil et al. [19] presented a really comprehensive review of simultaneous LS and scheduling problems, including sensible discussion of implementations of CLSP (big- bucket) and GLSP (small-bucket) in different industries. A systematic analysis of DLSPs for single-item and changes included correlation with all other decisions was proposed by Brahimi et al. [18].

Kopanos et al. [1] researched the issue of lot-sizing and scheduling in a yogurt production facility. They proposed a hybrid discreet/continuous-time MILP model with product family and a sequence-dependent setup time and costs. The issue in question focused primarily on the packaging level, while time and capability limitations were placed on pasteurization, homogenization, and fermentation processes. However, they considered that the scheduling problem only involves the packaging stage. In another article, Kopanos et al. [2] suggested a planning and scheduling model with capacity limitations. The model targets to reduce the total costs, including inventory, operating, and product family changeover costs.

Doganis and Sarimveis [9] suggested a MILP formulation for the packaging line with a sequence-dependent setup time and cost for a single machine, taking into consideration all typical limitations found in the scheduling of production (material balances, machine capacity, and inventory limits). The model, however, was restricted to a single production line. They modified their model to involve multiple parallel machines in another paper [10]. The new approach includes features that enable it to resolve manufacturing issues. However, it does not involve assumptions on multi-stage production and lacks some manufacturing features, including lifespan of products.

Marinelli et al. [12] suggested parallel packaging lines with sequence-independent setup time and cost and shared buffers for capacitated lot-sizing and scheduling model. Then to solve it, a two-step optimization decomposition process was applied.

Stefansdottir et al. [14] researched the issue of lot-sizing and scheduling issues to reduce the number and size of set-ups in the standard process-setting of cheese production in no-wait flow shops.

Kopanos et al. [17] proposed a multi-site, multi-product MILP model for the simultaneous systematic production and distribution through focusing on a hybrid discreet/continuous time frame in a semi-continuous food industry. The presented mathematical model’s difficulty arises within the incorporation of different approaches of modeling and even in specific processes of production and distribution.

Sel et al. [18] discussed the issue of the dairy sector concerning the embedded planning and scheduling of the set yogurt industry. The integrated problem was split into two sub-problems. For solving the sub-problems, heuristic decomposition was proposed.

2.2 Explicitly perishable feature and uncertainty in production planning and scheduling

Although shelf-life and life-time are significant main characteristics of perishable products, the perishability of food products, has not been explicitly addressed in previous papers. In this section we will review some works that explicitly addressed the perishability featured for products in production planning and scheduling. Amorim et al. [5] suggested two multi-objective methodologies for MTO (make to order) and MTO-MTS (hybrid make to order and make to stock) approaches for simultaneous lot-sizing and planning of product shelf-life and block planning. Non-dominated Sorting Genetic Algorithm II (NSGA II) was used to solve the issue.

Entrup et al. [6] also implemented three different production planning and scheduling models for perishable foods with objective functions that incorporate the shelf-life of food products. They also introduced an MILP formulation with a block planning method.

Bilgen and Celebi [8] gave rise to the stochastic features of the dairy manufacturing process sector and developed the shelf-life constraints of the MILP model to increase the freshness of the product. In addition, they developed a hybrid approach by integrating the MILP and simulation methods. Production time is defined to be a dynamic factor in the hybrid method and is iteratively modified by simulation and optimization model results.

Sarimveis and Doganis [11] suggested a new formulation by considering the cost of product life-span in the objective function. The extended model takes into account the shelf-life constraints and optimized the equilibrium between the profit-contributing factors and the cost factors through a minimization of the time between manufacture and distribution.

Bilgen et al. [13] proposed a novel method for a continuous, multi-stage, development planning issue that occurs in the dairy sector. The issue involves production features unique to the dairy sector, including shelf life, packaging rates, and storage intermediate.

Kopanos et al. [16] proposed a multi-product and multi-stage MILP model and a solution approach to solve the complicated problems in food industries' production scheduling with a short life-span of intermediate mixtures. The key characteristics of the suggested solution are the integrated production steps and the use of high valid integer breaks, choosing shorter computing cycles.

Kopanos et al. [15] developed a production planning and scheduling for a multi-stage, real-world food process with a restricted shelf-life of intermediate mixtures in the aging stage. They suggested an effective MIP on a continuous-time basis according to an appropriate sequencing decision-making modeling approach, integrated modeling of all stages of development, and implementation of a series of influential tightening restrictions to improve computational efficiency.

Although the majority of the previous studies on the modeling of lot-sizing and scheduling problems have explicitly addressed the perishability featured for products, but ignored the uncertainty of demand. The main motivation for a lot-sizing and scheduling problem in the food industry with uncertain demand is that within the deterministic demand, there will never be a spoiled product unless the minimum lot-sizes are very large compared to the demand orders. However, concerning the real-world challenges, the food industries face huge difficulties in decreasing the amount of wastage from non-selling products, whereas, only a limited number of researchers have considered the lot-sizing and scheduling problems with uncertain demand.

In many cases, there is inadequate information for decision-makers to adjust the demand probability distribution [22], which restricts the effectiveness of stochastic programming in cooperation with uncertainty. However, decision makers prefer to get an unknown parameter estimated on their own, which can be pessimistic, optimistic, and moderately optimistic to predict [32, 34]. Using fuzzy logic to overcome uncertainty relies on expert knowledge and perspectives is also advantageous. In a fuzzy programming model, there are different categories of fuzzy numbers most typically used, namely trapezoidal fuzzy numbers and triangular fuzzy numbers. In this analysis, we chose triangular fuzzy numbers to model uncertain demands.

Additionally, fuzzy possibility measure to build the fuzzy chance constraint has increasingly received the interest of many researchers in different areas such as biofuel supply chain (Tong K. et al., 2014 [32], meat supply chain (Mohammed, A. et al., 2017) [33], vehicle routing scheduling (Mousavi, S. M., et al. 2013) [34], inventory routing problem, Niakan, F., et al. 2015) [35], closed-loop supply chain network design (Torabi, S. A. et al., 2016) [36]. A fuzzy event will fail even though its possibility is 1; however, a fuzzy activity must hold when its credibility is 1 and it must fail when its credibility is 0. This is because of the fact that fuzzy measure of credibility is self-dual but fuzzy possibility evaluation is not [37]. Thus the fuzzy credibility metric is most acceptable for construction of restrictions on fuzzy chance and will be used in this research besides the use of possibility and necessity metrics.

2.3 Research gap

To the best of our knowledge, there is no study in the area of lot-sizing and scheduling problem that adopts possibalitic programming (PP) to deal with the uncertain demand. Hence, in this research, PP approach is integrated with fuzzy goal programming for integrating lot-sizing and scheduling problem for the food industry with demand uncertainty, which is typical in this industry, especially in the dairy and yogurt industry.

Overall, the review of literature indicates that there are some gaps in the yogurt industry in the field of lot-sizing and scheduling:

- Considering the lot-sizing and scheduling as a multi-objective problem in the yogurt industry, decision-makers often require two or more indicators of efficiency of the production as multi-objective.
- Considering this problem under demand uncertainty.
- Using the possibalitic approach to cope with uncertain parameters and to get a crisp model from the uncertain model.
- Applying fuzzy goal programming to address the suggested bi-objective model.

3. MATHEMATICAL FORMULATION

3.1. PROBLEM STATEMENT

The yogurt sector tends to operate in a competitive environment. The issue of production planning discussed in this work is relied on a case study of a yogurt firm. Milk follows a set of changes relevant to a certain total process in the dairy field. In a complicated process, the specific products are prepared, involving handling and preparation of raw milk, pasteurization, sterilization, fermentation, and packaging. Such manufacturing processes, also sometimes referred to as "continuous single-stage processes", are used to manufacture different perishable goods. Special care should be taken to achieve consistent quality, monitoring of allergens, batch quality control, and optimal product freshness. The company's main priority is to incorporate a solution able to enhance key efficiency measures, including total completion time minimization, which reflects on resource efficiency and cost minimization.

Units at the stage are non-identical in the sense that their applicability for handling goods as well as in their processing speeds are varied. The bottleneck of a yogurt manufacturing plant is the pasteurization, sterilization, fermentation, and packing processes mainly considering the low manufacturing speeds comparing to the flow rates previous phases and the design of the batch size of the process. It is important to tightly coordinate the production of intermediate and final products. Tank capacity, process time change, flow rate, and a variety of other technological restrictions must be considered for making a manufacturing process operable. Products are manufactured in lots and should be processed within the full capacity of the tank. Resources and tanks clean-up could be particularly complex. The shelf-life makes the manufacturing procedure a more inherent complexity. The assumed manufacturing process has a certain unique characteristic that needs serious attention in the context of production planning. The main variables to be determined in this process are the amount produced within every period of time, the quantity of final products held in inventory, and the completion time. The objectives are to minimize the total cost such as setup, inventory holdings and production costs, and minimizing the total completion time.

In order to produce and package multiple items in processing operations, a variety of parallel processing equipment (mixing tanks and packing lines) are available. Many technical restrictions should comply; each resource is unable to handle more than one product at a time, but different products can share a similar processing resource and the same product can be processed in parallel by different resources. The manufacturing resource is initially modified to manufacture yogurt items in a specified package with a specified taste. Stopping the production and making possible modifications are essential in order to manufacture another yogurt product with a different pack size and flavor. Hence, different costs and operational restrictions should be taken into consideration in designing the production schedule.

Due to the use of tanks for fermentation of different dairy products, the fermentation processes are only taken into account by restricting the capacity and implementing minimum lot sizes for the packaging lines. In addition, product distribution is not considered in the models because it is mostly done out by supermarket organizations [6].

To formulate the problem, we first propose inventory and time constraints in a production environment, including semi-continuous flow lines, multiple feed lines (as parallel shared common resources), and series-parallel machines (fillers). The multi-objective function involves the optimization of both total production costs and completion time.

In addition the proposed mathematical model enjoys the following features:

- Tanks could feed any production line without additional costs.
- Manufacturing system includes parallel and unrelated lines.
- To change the size of the package in the production lines, the set-up costs are endorsed.
- The manufacturing line is a continuous stage, neglecting the middle stages and the work in progress.
- Set-up times and costs are sequence-dependent for the production lines.
- Shelf-life issues are explicitly included in the model.
- Owing to perishable features, the handling of the product inventory is based on FIFO.
- Like other food industries, in the yogurt production line, we should consider the clean in place (CIP) that is run after each lot
- The time horizon for scheduling is one day.
- The lower bound of lot sizes is specified by profits, and the upper bound of lot sizes is related to the fermentation capacity.

3.2 Modelling approach

In this part, we present an MILP formulation with demand uncertainty. The model specifies optimal schedules with the goal of minimizing the production makespan and the overall cost of production (holding, changeover, and operation) as a measure for efficiency of the production, assigning of items to the equipment, ordering of the manufacturing products on each machine, and the quantity of production on each product. The specific notation required for the models is described below.

Sets

$i \in I$: Intermediate mixture

$p, k \in P$: Finished products

$j \in J$: Packaging unit

$t \in T$: Scheduling time periods

Parameters

$d_{p,t}$:Demand of product p at period t

$h_{p,t}$: Inventory cost for product p at period t

$ch_{p,k,j,t}$: Sequence-dependent setup cost between both products (p,k) in packaging unit j at time t

Cf : Fermentation cost

$cp_{p,j,t}$: Operating cost for packaging product p in unit j period t

$T_{p,j}$: Time required for preparing the product p on unit j

$sd_{p,k,j}$: Sequence-dependent changeover time between product p and product k in equipment unit j at time t

$\emptyset_{p,t}$: The necessary time quality control

$rate_{p,j}$: Packaging rate for product p at packaging unit j

$QI_{i,t}^{min}$:Minimum processing lot-size

$QI_{i,t}^{max}$: Maximum processing capacity

$QL_{p,j}^{min}$,$QL_{p,j}^{max}$: Minimum and maximum production amount of products

$Tp_{p,j,t}^{max}$,$Tp_{p,j,t}^{min}$: Maximum and minimum run time for product p in unit j at time t

$I_{p,t}^{safty}$: Safety stock for product p at period t

L_t: Scheduling horizon

I_0 : Initial inventory

sl: Lifetime of products

M: A very large number

Decision variables

$Q_{p,j,t}$: Packaging quantity of each product

$I_{p,t}$:The amount of inventory of each product at the end of the day

$Tp_{p,j,t}$: Packaging time of product p in unit j at time t

$C_{p,j,t}$: Completion time of product p in unit j at time t

$Y_{p,j,t}$:Binary variables showing whether product p has been allocated to unit j at time t

$X_{p,k,j,t}$:Binary variables indicating whether product p is handled completely before k, when two products are allocated at the same time t to the same unit j

$V_{i,t}$: Binary variables indicating the assignments for producing intermediate product i at processing cycle t on the production unit

3.3 Mathematical formulation

In this part, we propose a bi-objective MILP method. The management of product inventory is focused on FIFO because of the characteristics of perishable products. To clarify, it is assumed that the first items produced are the first ones sold. As a measure of the efficiency of production, the proposed model aims to reduce the total production costs, which include change-over, inventory, operational and fermentation costs and minimization of the makespan of production.

The objective functions:

$$Minimize \sum_p \sum_t h_{p,t}.I_{p,t} + \sum_p \sum_k \sum_j \sum_t X_{p,k,j,t}.ch_{p,k,j,t} + \sum_p \sum_j \sum_t cp_{p,j,t}.Tp_{p,j,t} + \sum_i \sum_t \text{Cf}.\ V_{i,t} \quad (1)$$

$$Minimize\ C_{\max} \quad (2)$$

Constraints:

Timing Constraints:

$$Tp_{p,j,t} = \frac{Q_{p,j,t}}{rate_{p,j}} \quad \forall p,j,t \quad (3)$$

$$Tp_{p,j,t}^{min}.Y_{p,j,t} \leq Tp_{p,j,t} \leq Tp_{p,j,t}^{max}.Y_{p,j,t} \quad \forall p,j,t \quad (4)$$

$$C_{p,j,t} - Tp_{p,j,t} \geq (\emptyset_{p,t} + T_{p,j}).Y_{p,j,t} + \sum_{k\neq p} sd_{p,k,t}.X_{p,k,j,t} \quad \forall p,j,n \quad (5)$$

$$C_{p,j,t} \geq L_t.Y_{p,j,t} \quad \forall p,j,t \quad (6)$$

Time-dependent changeovers:

$$C_{k,j,t} - Tp_{k,j,t} \geq C_{p,j,t} + sd_{p,k,j} - M.(1 - X_{p,k,j,t}) - \text{M}.(2 - Y_{p,j,t} - Y_{K,j,t}) \quad \forall p,k,j,t \quad (7)$$

$$C_{p,j,t} - Tp_{p,j,t} \geq C_{k,j,t} + sd_{p,k,j} - M.X_{p,k,j,t} - \text{M}.(2 - Y_{p,j,t} - Y_{K,j,t}) \quad \forall p,k,j,t \quad (8)$$

Allocation and sequencing constraints:

$$\sum_{k} X_{p,k,j,t} \leq Y_{p,j,t} \qquad \forall p,t,j \in J \tag{9}$$

$$\sum_{\mathrm{k}} X_{k,p,j,t} \leq Y_{p,j,t} \qquad \forall p,t,j \in J \tag{10}$$

$$\sum_{j} Y_{p,j,t} \leq 1 \qquad \forall p,t \tag{11}$$

$$\sum_{p}\sum_{k} X_{p,k,j,t} + 1 = \sum_{p} Y_{p,j,t} \quad \forall t,j \tag{12}$$

$$V_{i,t} \geq \sum_{j} Y_{p,j,t} \qquad \forall i,p,t \tag{13}$$

Capacity constraints:

$$QL_{p,j}^{min}.Y_{l,j,t} \leq Q_{p,j,t} \leq QL_{p,j}^{max}.Y_{p,j,t} \qquad \forall p,t,j \in J_l \tag{14}$$

$$QI_{p,j}^{min}.V_{i,t} \leq \sum_{p}\sum_{j} Q_{p,j,t} \leq QI_{p,j}^{max}.V_{i,t} \qquad \forall i,t \tag{15}$$

Mass balance constraints:

$$I_{p,t} \geq I_0 + \sum_{j} Q_{p,j,t} + d_{p,t} \qquad \forall p,t: t = 1 \tag{16}$$

$$I_{p,t} \geq I_{p,t-1} + \sum_{j} Q_{p,j,t} + d_{p,t} \qquad \forall ,t: t > 1 \tag{17}$$

$$I_{p,t} \leq \sum_{t'=t}^{t+sl} d_{p,t'} \qquad \forall p,t \leq T - sl \tag{18}$$

$$I_{p,t} \leq \sum_{t'=t+1}^{T} d_{p,t'} \qquad \forall p,t > T - sl \tag{19}$$

$$I_{p,t} \geq I_{p,t}^{safty} \qquad \forall l,t \qquad (20)$$

In Constraint (3), $Tp_{p,j,t}$ means the packaging time (the processing time) of product p in unit j at time t which is equal to the packed quantity $Q_{p,j,t}$ of product p at the same packaging unit divided by the packaging rate $rate_{p,j}$ of product p. Soman et al. [24] mentioned that in the food industry, which has high capacity of utilization, the production rate $rate_{p,j}$ is considered fixed, where reducing the production rate can affect the quality of the products. Constraint (4) enforces the upper and lower bounds on the packaging time $Tp_{p,j,t}$. Constraints (5) imposes that the start time $(C_{p,j,t} - Tp_{p,j,t})$ of product p on unit j and time t has to be greater than the necessary time quality control time $\emptyset_{p,t}$, plus the time required for preparing the product p on unit j and fermentation recipe and maintenance or other technical issues, and plus the changeover time $sd_{p,k,t}$ for transferring the production to item k. Constraint (6) implies the upper time of the completion time; it further ensures that the completion time is less than the scheduling horizon. Constraints (7, 8) determine the beginning times on unit j, taking into consideration sequence-dependent setup times. By adjusting the parameter $sd_{p,k,j}$, the setup time can be changed to the successor or predecessor dependent setups. It also guarantees a workable timing of the products allocated to the same unit of processing. Constraints (9, 10) indicate that if product p is assigned to unit j at time t, at most, one product k is handled just after or/and before it. Constraint (11) forbids the batch splitting for packaging operations. Constraint (12) guarantees that the overall number of effective sequencing variables $X_{p,k,j,t}$ should be greater than the overall number of effective assignment binary variables, $Y_{p,j,t}$ minus one. Constraint (13) represent the interaction between packaging units and fermentation phase. Capacity constraint (15) imposes a product's packaged quantity to be minimal than its corresponding upper packaging rate $QL_{p,j}^{max}$ and larger than its lower packaging rate $QL_{p,j}^{min}$. Constraint (16) ensures maximum and minimum lot sizes for producing intermediate product. Constraints (17, 18) guarantee the inventory equilibrium. The relationship between the level of demand and inventory considering the life-span of the product is demonstrated by Constraint (19, 20). The lower bound, which imposes that the inventory should be larger than the safety stock at the end of work day, is shown by Constraint (21).

3.4 Problem with fuzzy demands

If the demand for different products is uncertain and imprecise, we propose modeling it using a fuzzy theoretical approach, and utilizing a fuzzy set $\widetilde{d_{p,t}}$, $p =$

$1, \dots, N,$ to define demand. The mathematical programming method is the same as above, except for the inventory constraints (16, 17, 18, and 19). The following fuzzy constraints for the fuzzy demands for product p must be considered instead of the crisp inventory constraints:

$$I_{p,t} \geq I_0 + \sum_{l} \sum_{j} Q_{l,j,t} + \widetilde{d_{p,t}} \qquad \forall p, t: t = 1 \qquad (21)$$

$$I_{p,t} \geq I_{p,t-1} + \sum_{l} \sum_{j} Q_{l,j,t} + \widetilde{d_{p,t}} \qquad \forall, t: t > 1 \qquad (22)$$

$$I_{p,t} \leq \sum_{t'=t}^{t+sl} \widetilde{d_{p,t'}} \qquad \forall p, t \leq T - sl \qquad (23)$$

$$I_{p,t} \leq \sum_{t'=t+1}^{T} \widetilde{d_{p,t'}} \qquad \forall p, t > T - sl \qquad (24)$$

4. Solution approach

Many strategies have been used to handle uncertainties in optimization problems. Fuzzy Theory is among the most commonly used strategies. In particular, the model of fuzzy possibilistic programming is widely applied, which is a strong mathematical optimization technique to be utilized under uncertainty to solve optimization issues. Furthermore, by using this method, as compared to the deterministic formulation, the problem size stays unchanged, meaning that the computational complexity of the issue does not rise [25]. Since this strategy adds no constraint to the problem model; therefore, it is convenient for NP-hard issues [27].

The use of fuzzy planning methods to deal with multi-objective issues by taking into account the problems of uncertainty is suggested [29]. In certain multi-objective issues, the primary aim is to accomplish the collection of objectives by decision makers. In our problem, the decision-maker would like to achieve goals relating to cost and completion time. Hence, we suppose that it is beneficial to use goal programming approaches. Therefore, here, a goal programming and a fuzzy possibilistic programming are proposed to obtain the goals' group out under uncertainty in a multi-objective issue.

4.1 Step one: Defining goals

DMs define the goals of the issue in this phase. To do so, the optimum value of each objective function is calculated by solving the objective functions separately. Then, based on the optimal values, DMs will decide the goals of each problem. These objectives, which are not restricted to the objective functions, can be specified for all restrictions. For instance, it is presumed in the suggested model that the optimal values of the individually solved for the first and second objective functions are Z_1^* and Z_2^*, respectively. If the goal relating to the first objective function is $Goal_1$ and to the second objective function is $Goal_2$, the two proposals that follow are valid:

$$Z_1^* \leq Goal_1 , Z_2^* \leq Goal_2$$

4.2 Step 2: goal programming approach

In the present stage, the goal programming approach is described by providing a group of goals for the issue. The model is as shown in Equation 25:

$$\begin{aligned} & Min\ D_1^+ \\ & Min\ D_2^+ \\ st: & \\ & Z_1 - D_1^+ + D_1^- = Goal_1 \\ & Z_2 - D_2^+ + D_2^- = Goal_2 \\ & \text{model } constraints \end{aligned} \quad (25)$$

Where, D_1^+ and D_2^+ are positive deviations, and D_1^- and D_2^- represent negative deviations, respectively, from the first and second goals. The model constraints comprise of the constraints of the suggested system. The two objective functions in the problem under study are minimization functions. Then the positive deviations from the goals should be minimized.

4.3 Step 3: Fuzzy possibilistic programming

For modeling the fuzzy demand, we suggest considering the possibility that the demand for all products to a certain degree is less or equal to the inventory. This contributes to a crisp equivalent model. The approach of handling the fuzzy constraints is close to the programming of chance constraints in stochastic optimization. It is supposed that the fuzzy parameters can be managed with at least the credibility, possibility, or necessity, known as the “degree of confidence”. If the demand is not precisely defined, we suggest trying to find a solution that needs at least a certain degree of possibility $\alpha \in [0,1]$. This degree must be decided in advance by the decision maker. Furthermore, the determination of a certain degree of necessity that the demand can be fulfilled is an even greater condition. In addition, we can consider the fuzzy case's credibility, which is determined as the average of its necessity and possibility:

$$Pos/Nes/Cr\left(I_{p,t} \geq I_0 + \sum_l \sum_j Q_{l,j,t} - \widetilde{d_{p,t}}\right) \geq \alpha \quad \forall p,t: t=1; \alpha \in [0,1] \qquad (26)$$

$$Pos/Nes/Cr\left(I_{p,t} \geq I_{p,t-1} + \sum_l \sum_j Q_{l,j,t} - \widetilde{d_{p,t}}\right) \geq \alpha \quad \forall p,t: t>1; \alpha \in [0,1] \qquad (27)$$

$$Pos/Nes/Cr\left(I_{p,t} \leq \sum_{t'=t}^{t+sl} \widetilde{d_{p,t'}}\right) \geq \alpha \qquad \forall p, t \leq T - sl\, ; \alpha \in [0,1] \qquad (28)$$

$$Pos/Nes/Cr\left(I_{p,t} \leq \sum_{t'=t+1}^{T} \widetilde{d_{p,t'}}\right) \geq \alpha \qquad \forall p, t > T - sl\, ; \alpha \in [0,1] \qquad (29)$$

In order to calculate, let us initially assume that the necessity and possibility and credibility of a triangular fuzzy number may be greater or equal to zero as follows:

$$pos(\tilde{\xi} \geq 0) = pos_{\tilde{\xi}}(\{x|x \geq 0\}) = sup\mu_{\tilde{\xi}}(\{x|x \geq 0\}) \qquad (30)$$

$$pos(\tilde{\xi} \leq 0) = pos_{\tilde{\xi}}(\{x|x \leq 0\}) = sup\mu_{\tilde{\xi}}(\{x|x \leq 0\}) \qquad (31)$$

$$Nec(\tilde{\xi} \geq 0) = Nes_{\tilde{\xi}}(\{x|x \geq 0\}) = 1 - sup\mu_{\tilde{\xi}}(\{x|x < 0\}) \qquad (32)$$

$$Nec(\tilde{\xi} \leq 0) = Nes_{\tilde{\xi}}(\{x|x \leq 0\}) = 1 - sup\mu_{\tilde{\xi}}(\{x|x > 0\}) \qquad (33)$$

$$cr(\tilde{\xi} \geq 0) = \frac{1}{2}\left[pos(\tilde{\xi} \geq 0) + Nec(\tilde{\xi} \geq 0)\right] \qquad (34)$$

$$cr(\tilde{\xi} \leq 0) = \frac{1}{2}\left[pos(\tilde{\xi} \leq 0) + Nec(\tilde{\xi} \leq 0)\right] \qquad (35)$$

The possibility (Pos), necessity (Nec), and credibility (Cr) can be calculated using the following formulas for a triangular fuzzy number $\tilde{\xi} = (\bar{\xi}, \hat{\xi}, \underline{\xi})$ with $\hat{\xi} \neq \bar{\xi}$ and $\hat{\xi} \neq \underline{\xi}$:

$$pos(\tilde{\xi} \geq 0) = \begin{cases} 1 & \hat{\xi} \geq 0, \\ \dfrac{\underline{\xi}}{\underline{\xi} - \hat{\xi}} & \hat{\xi} \leq 0 \leq \underline{\xi} \\ 0 & \hat{\xi} \leq 0 \end{cases} \tag{36}$$

$$pos(\tilde{\xi} \leq 0) = \begin{cases} 0 & \hat{\xi} \leq 0, \\ \dfrac{-\underline{\xi}}{\underline{\xi} - \hat{\xi}} & \hat{\xi} \leq 0 \leq \underline{\xi} \\ 1 & \hat{\xi} \geq 0 \end{cases} \tag{37}$$

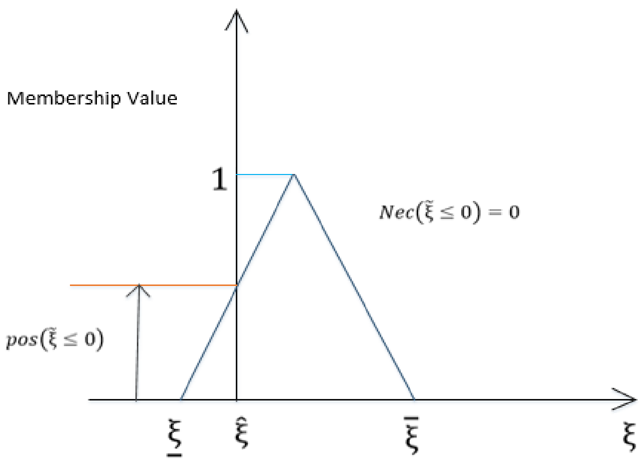

Fig. 1. $pos(\tilde{\xi} \leq 0)$ for a triangular fuzzy number $\tilde{\xi}$

$$Nec(\tilde{\xi} \geq 0) = \begin{cases} 1 & \bar{\xi} \geq 0, \\ \dfrac{\hat{\xi}}{\hat{\xi} - \bar{\xi}} & \bar{\xi} < 0 \leq \hat{\xi} \\ 0 & \hat{\xi} < 0 \end{cases} \tag{38}$$

$$Nec\left(\tilde{\xi} \geq 0\right) = \begin{cases} 0 & \bar{\xi} \leq 0, \\ \dfrac{-\widehat{\xi}}{\hat{\xi} - \bar{\xi}} & \bar{\xi} \leq 0 \leq \hat{\xi} \\ 1 & \hat{\xi} \geq 0 \end{cases} \tag{39}$$

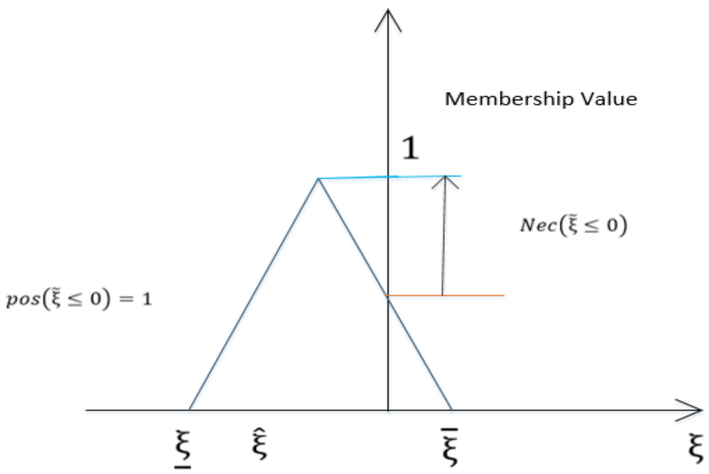

Fig. 2. $Nec\left(\tilde{\xi} \leq 0\right)$ for a triangular fuzzy number $\tilde{\xi}$

Credibility (Cr) of fuzzy events:

$$cr\left(\tilde{\xi} \leq 0\right) = \begin{cases} \dfrac{\bar{\xi}}{\bar{\xi} - \hat{\xi}} & \hat{\xi} \leq 0 \leq \hat{\xi} \\ 0 & \hat{\xi} = 0 \\ \dfrac{-\underline{\xi}}{\underline{\xi} - \hat{\xi}} & \hat{\xi} \leq 0 \leq \underline{\xi} \\ 1 & otherwise \end{cases} \tag{40}$$

$$cr(\tilde{\xi} \geq 0) = \begin{cases} \dfrac{-\bar{\xi}}{\bar{\xi} - \hat{\xi}} & \hat{\xi} \leq 0 \leq \hat{\xi} \\ 1 & \hat{\xi} = 0 \\ \dfrac{\underline{\xi}}{\underline{\xi} - \hat{\xi}} & \hat{\xi} \leq 0 \leq \underline{\xi} \\ 0 & otherwise \end{cases} \tag{41}$$

$$pos\{\xi \leq 0\} \geq \alpha \rightarrow 0 \geq (1-\alpha)\hat{\xi} + \alpha\underline{\xi} \tag{42}$$

$$pos\{\xi \geq 0\} \geq \alpha \rightarrow 0 \leq \alpha\hat{\xi} + (1-\alpha)\underline{\xi} \tag{43}$$

$$Nes\{\xi \leq 0\} \geq \alpha \rightarrow 0 \geq (1-\alpha)\bar{\xi} + \alpha\,\hat{\xi} \tag{44}$$

$$Nes\{\xi \geq 0\} \geq \alpha \rightarrow 0 \leq \alpha\bar{\xi} + (1-\alpha)\hat{\xi} \tag{45}$$

$$Cr\{\xi \leq 0\} \geq \alpha \rightarrow 0 \geq (2-2\alpha)\hat{\xi} + (2\alpha-1)\bar{\xi} \tag{46}$$

$$Cr\{\xi \geq 0\} \geq \alpha \rightarrow 0 \leq (2\alpha-1)\underline{\xi} + (2-2\alpha)\hat{\xi} \tag{47}$$

Therefore, the condition for the probability that the inventory is appropriate in the latter model can be calculated as follows:

$$Pos/Nes/Cr\left(I_{p,t} \geq I_0 + \sum_l \sum_j Q_{l,j,t} - \widetilde{d_{p,t}}\right) \geq \alpha \Leftrightarrow pos\left(I_{p,t} - I_0 - \sum_l \sum_j Q_{l,j,t} + \widetilde{d_{p,t}} \geq 0\right) \geq \alpha \quad \forall p,t: t = 1; \alpha \in [0,1] \tag{48}$$

$$Pos/Nes/Cr\left(I_{p,t} \geq I_{\mathrm{p,t}} + \sum_{l}\sum_{j} Q_{l,j,t} - \widetilde{d_{p,t}}\right) \geq \alpha$$
$$\Leftrightarrow\ pos\left(I_{p,t} - I_{\mathrm{p,t}} - \sum_{l}\sum_{j} Q_{l,j,t} + \widetilde{d_{p,t}} \geq 0\right)$$
$$\geq \alpha \quad \forall p,t{:}\, t > 1;\alpha \in [0,1] \qquad (49)$$

$$Pos/Nes/Cr\left(I_{p,t} \leq \sum_{t'=t}^{t+sl} \widetilde{d_{p,t'}}\right) \geq \alpha \ \Leftrightarrow\ pos\left(I_{p,t} - \sum_{t'=t}^{t+sl} \widetilde{d_{p,t'}} \leq 0\right)$$
$$\geq \alpha \qquad \forall p,t \leq T - sl\,;\alpha \in [0,1] \qquad (50)$$

$$Pos/Nes/Cr\left(I_{p,t} \leq \sum_{t'=t+1}^{T} \widetilde{d_{p,t'}}\right) \geq \alpha$$
$$\Leftrightarrow\ pos\left(I_{p,t} - \sum_{t'=t+1}^{T} \widetilde{d_{p,t'}} \leq 0\right) \qquad \forall p,t > T - sl\,;\alpha$$
$$\in [0,1] \qquad (51)$$

4.4 Step 4: Deterministic equivalent crisp model

If all demands $\widetilde{d_{p,t}} = (\overline{d_{p,t}}, \widehat{d_{p,t}}, \underline{d_{p,t}})$, $p = 1, \ldots, P$ are triangular fuzzy numbers, then the following constraints can be modeled as crisp alternatives for the fuzzy constraint for $\alpha > 0$:

$$Pos\big(served\widetilde{d_{p,t}}\big) \leq \alpha \Leftrightarrow I_{p,t}$$
$$\geq I_0 + \sum_{l}\sum_{j} Q_{l,j,t} - \left((1-\alpha)\widehat{d_{p,t}} + \alpha\underline{d_{p,t}}\right) \qquad (52)$$

$$I_{p,t} \geq I_{\mathrm{p,t-1}} + \sum_{l}\sum_{j} Q_{l,j,t} - \left((1-\alpha)\widehat{d_{p,t}} + \alpha\underline{d_{p,t}}\right) \qquad (48)$$

$$I_{p,t} \leq \sum_{t'=t}^{t+sl} \overline{d_{p,t}} + \alpha\left(\sum_{t'=t}^{t+sl} \widehat{d_{p,t}} - \sum_{t'=t}^{t+sl} \overline{d_{p,t}}\right) \qquad (53)$$

$$I_{p,t} \leq \sum_{t'=t+1}^{T} \overline{d_{p,t}} + \alpha\left(\sum_{t'=t+1}^{T} \widehat{d_{p,t}} - \sum_{t'=t+1}^{T} \overline{d_{p,t}}\right) \qquad (54)$$

$$Nes\left(serve\ \widetilde{d_{p,t}}\right) \le \alpha \Leftrightarrow I_{p,t} \ge I_0 + \sum_l \sum_j Q_{l,j,t} - \left(\alpha \widehat{d_{p,t}} + (1-\alpha)\underline{d_{p,t}}\right) \quad (56)$$

$$I_{p,t} \ge I_{\mathrm{p,t-1}} + \sum_l \sum_j Q_{l,j,t} - \left(\alpha \widehat{d_{p,t}} + (1-\alpha)\underline{d_{p,t}}\right) \quad (57)$$

$$I_{p,t} \le (1-\alpha) \sum_{t'=t}^{t+sl} \widehat{d_{p,t}} + \alpha \sum_{t'=t}^{t+sl} \overline{d_{p,t}}) \quad (58)$$

$$I_{p,t} \le (1-\alpha) \sum_{t'=t+1}^{T} \widehat{d_{p,t}} + \alpha \sum_{t'=t+1}^{T} \overline{d_{p,t}}) \quad (60)$$

$$Cr\left(serve\widetilde{d_{p,t}}\right) \le \alpha \Leftrightarrow I_{p,t} \ge I_0 + \sum_l \sum_j Q_{l,j,t} - \left((2-2\alpha)\widehat{d_{p,t}} + (2\alpha-1)\underline{d_{p,t}}\right) \quad (61)$$

$$I_{p,t} \ge I_{\mathrm{p,t-1}} + \sum_l \sum_j Q_{l,j,t} - \left((2-2\alpha)\widehat{d_{p,t}} + (2\alpha-1)\underline{d_{p,t}}\right) \quad (62)$$

$$I_{p,t} \le (2\alpha-1) \sum_{t'=t}^{t+sl} \overline{d_{p,t}} + (2-2\alpha) \sum_{t'=t}^{t+sl} \widehat{d_{p,t}} \quad (63)$$

$$I_{p,t} \le (2\alpha-1) \sum_{t'=t+1}^{T} \overline{d_{p,t}} + (2-2\alpha) \sum_{t'=t+1}^{T} \widehat{d_{p,t}} \quad (64)$$

4.5 Step 5: Equivalent single objective model

There are many approaches in the literature to solve multi-objective linear programming problems (MOLP); however, a fuzzy programming method has been utilized to achieve a maximum degree of satisfaction with all objective functions by a wide variety of investigators. The primary benefit of this technique is that it enables the decision maker to find the optimum results based on his/her preferences or the proportional significance of each target. Zimmermann [29] was the first researcher to suggest a fuzzy method to solve the MOLP with a max-min approach.

Then Lai and Hwang [30] suggested an improved min-max method to overcome the weakness of the approach through providing more unique and reliable solutions.

Since the objective of programming is to minimize the unacceptable deviations from the objectives defined by the decision makers, to this end, the following membership function is applied:

$$\mu_{D_i}^{Min}(x) = \begin{cases} 1 & D_i(x) > UD_i \\ 0 & D_i(x) < LD_i \\ \frac{UD_i - D_i(x)}{UD_i - LD_i} & LD_i \leq D_i(x) \leq UD_i \end{cases} \quad (65)$$

Where, UD_i and LD_i are the upper and lower levels of unwanted deviations from the goal i, respectively.

The membership function of unwanted deviations from goal *i* is indicated by $\mu_{D_i}^{Min}$. As a result, the following fuzzy single objective formation is acquired:

$$\begin{gathered} Max\ \varphi \\ St: \\ \varphi \leq \mu_{d_i}^{Min}(x) \quad (66) \\ Z_i - D_i^+ + D_i^- = Goal_i \\ model\ constraints \end{gathered}$$

Based on the above constraint, the suggested mathematical method can be developed as below:

$$\begin{gathered} Max\ \varphi \\ St: \\ \varphi \leq \frac{UD_1 - D_1(x)}{UD_1 - LD_1} \\ \varphi \leq \frac{UD_2 - D_2(x)}{UD_2 - LD_2} \quad (67) \end{gathered}$$

$$Z_1 - D_1^+ + D_1^- = Goal_1$$

$$Z_2 - D_2^+ + D_2^- = Goal_2$$

$$model\ constraints$$

Accordingly, we developed a fuzzy goal programming approach with a possibilistic constraint to solve the proposed model, which allows DMs for making better deciding and sensitive analyses. The proposed methodology to solve the problem is shown in Fig. 3.

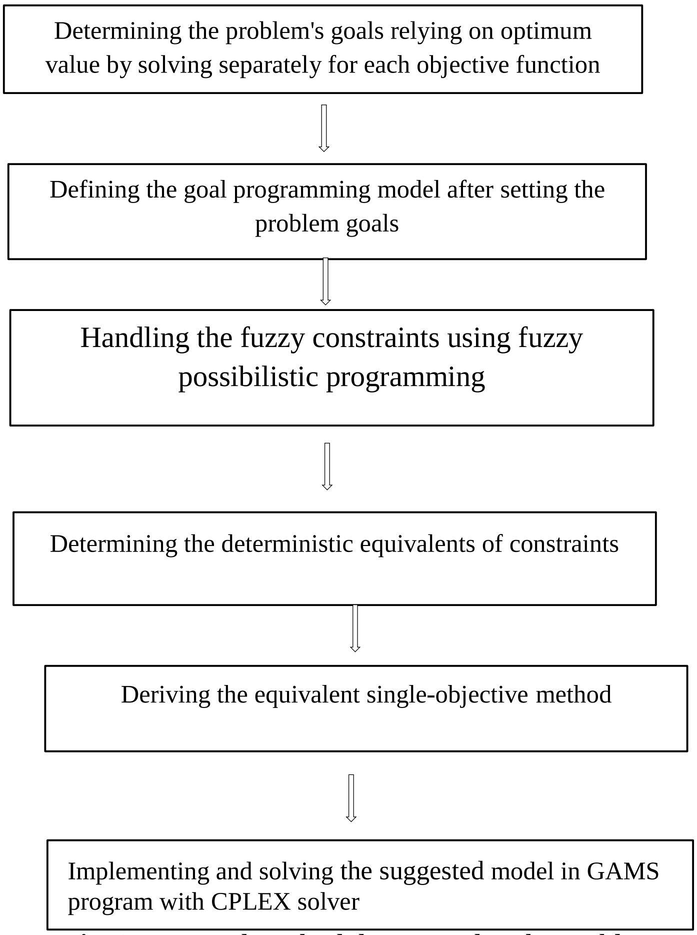

Fig. 3 Proposed methodology to solve the problem

5. **Case study**

To illustrate the numerical validation and applicability of the suggested model, the production planning and scheduling problems faced by a small local yoghurt factory are presented. Therefore, as an explanatory example, the set form of yoghurt production process is handled. The manufacturing line of yogurt is comprised of a collection of cooling tanks (set yogurt) and straight forms of three parallel filling and packing systems.

The scheduling is carried out across an hourly time period in the case study (i.e., 18h). For yoghurt production, the short-term scheduling time period is typically 1 week. The standards for seven product categories are fulfilled within normal operating hours. There is a minimum lot size of 1,200 l for the processing phase, and the sequence-dependent setup times range from 0.5 to 1.5 h for transformation between the product categories. In a packing unit, the minimum production quantity of any product type is 150 kg. The capability of the tank restricts the overall output quantity of the product category in each unit of packaging. The packaging speed of each packaging line and other machines' data are shown in Table 1. In addition, the initial inventory and holding costs are given in Table 2.

Table 1 : Machine data

	Unit 1	Unit 2	Unit 3
Set-up time (minutes)	137	195	80
Set-up cost ($)	20	28	12
Packing rate (litres/hours).	4615	2640	2013

Table 2: Products data

Product	Initial inventory	Inventory cost ($)
P1	4,000	0.025
P2	2,000	0.075
P3	750	0.125
P4	500	0. 25
P5	1,700	0.025
P6	1,250	0.075
P7	1,600	0.125

To discover the pattern of demand for yogurt products in the case under study, the data regarding the monthly sales of yogurt products Dairy Business were collected over 60 months from 2016 to 2019. The pattern of demand over this period is illustrated in Figure 4. As shown, there is no specific pattern of demand for dairy products.

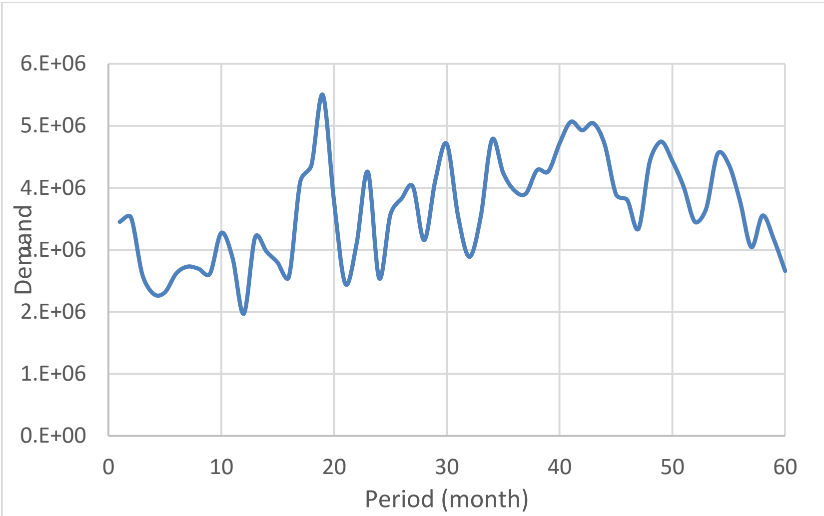

Fig. 4 Total demand for dairy products

Since adequate data are not always available to forecast uncertain parameters, the choice of a fuzzy set theory is more rational and compelling to convey the complexity of expert expertise. Most of the significance in the theory of the fuzzy system lies in the portrayal of complexity in the human cognitive process [26].

The demand, as a fuzzy parameter, is represented in this paper as a triangular membership function. We consider $(\bar{\xi}, \hat{\xi}, \underline{\xi})$ where, $\bar{\xi}$ is the most pessimistic, $\hat{\xi}$ is the most likely and $\underline{\xi}$ is the most optimistic value for characterizing the fuzzy number of triangle $\tilde{\xi}$. These values must be estimated for each fuzzy parameter using the approach suggested by Lai and Hwang [28]. First, according to the uniform distribution, the most possible value for each uncertain parameter is assigned randomly. Then the most pessimistic $\bar{\xi}$ and the most optimistic $\underline{\xi}$ values of a fuzzy number ξ are obtained as $\bar{\xi} = (1 - r_1)\hat{\xi}, \underline{\xi} = (1 + r_1)\hat{\xi}$, respectively, where (r_1, r_2) are two numbers randomly generated according to the uniform distribution (0.1, 0.3).

6. Experimental results

This part is devoted to discussing the research outcomes. The suggested mathematical model is implemented in GAMS software and solved with CPLEX solver by introducing the case data and running it on a 2.2 GHz personal computer core i7 with 8.0 GB RAM. The resulting integrated model has 1,842 continuous and 1,356 binary variables, and 1,934 constraints. According to DMs, the possibility of meeting demand constraints at the degree of α=0.7, the first and second goals were deemed 14(hours) in each day for reason related with consuming energy, 5000($) in horizon period, respectively.

The crisp equivalent fuzzy goal model for a requested possibility α = 0.7 to meet the demand was solved, and the obtained results as well as the model statistics are displayed in Table 3. Compared to the deterministic model, the size of the problem stays constant, which demonstrates the excellent computational efficiency of the possibilistic programming. The probability measure indicates an optimistic attitude of the decision maker in relation to what we explained in the possibilistic programming, while the necessity measure represents a pessimistic attitude. In the model of possibility, demand for the product has a tendency in a broader range, while the demand tends to have a limited range in the necessity model. The outcome demonstrates that beneath 0.7 possibility, the total cost is 7255.912 and maximum completion time is 14.760, whereas beneath 0.7 certainty level, the total cost and the maximum completion time are 4769.915 and 14.232, respectively.

The measure of credibility is the average of necessity and possibility; however, it appears to be more pessimistic. The outcome shows that beneath 0.7 credibility degree, the total cost is 5719.925 and the maximum completion period is 14.636, which is a trade-off between the two models, but close to the necessity result. Tables 4 and 5 introduce the production schedule and the inventory of products, respectively. A total cost breakdown is shown in Figure 5.

Table 3: Model statistics.

Model	**Total cost**	**Maximum completion time**	**Time (s)**	**Number of binary variables**	**Number of continuous variables**	**Number of constraints**
Pos-model	7255.912	14.760	15.13	1,356	1,842	1,934
Nes-model	4769.915	14.232	55.38	1,356	1,842	1,934
Cr-model	5719.925	14.636	39.94	1,356	1,842	1,934

Table 4: Production Schedule

Product	**Package unit**	**Production periods (days)**					
		n0	**n1**	**n2**	**n3**	**n4**	**n5**
p1	j2	0	9163.505	0	0	0	0
p2	j2	0	0	3660.271	0	0	0
p3	j1	0	0	5380.894	5400	0	0
p3	j3	5109.41	5760	0	0	5760	5760
p4	j2	0	0	0	9961.461	10260	9326.638
p5	j2	6890.59	0	8602.727	7354.377	7055.838	7989.2
p6	j3	0	0	5760	3586.364	3187.295	0
p7	j3	0	0	5760	3328.416	0	0

Table 5: Inventory of product

Product	Production periods (days)				
	n0	n1	n2	n3	n4
p1	0	7343.015	0	0	0
p2	0	0	3353.984	0	0
p3	5109.41	211.5289	0	1098.959	0
p4	0	0	0	5060.161	4114.008
p5	6890.59	0	0	0	4127.831
p6	0	0	4038.942	1626.078	1470.515
p7	0	0	3421.027	1113.249	0

Fig. 5. Cost comparative analysis

7. Analyzing sensitivity

We carried out a sensitivity analyses with respect to α value in order to support our argument on the benefits of the suggested approach to provide flexibility in decision making. Six different α values (0.1, 0.3, 0.5, 0.7, 0.9, and 1.0) were used to represent six different decision-condition scenarios. As shown in Tables (6, 7, 8), the change in α value influences the maximum division from goal (φ), output results and objective values. The relationship between α value and (φ) is positively correlated to possibility and necessity measures. That is, the φ value rises from 0.499 to 0.541 when α value increases from 0.7 to 1.0 when applying the possibility measure. Also the φ value increases from 0.545 to 0.586, when α value increases from 0.1 to 1.0 when applying the necessity measure.

The relationship between α value and (φ) is negatively correlated to credibility measure. That is, the φ value decreases from 0.558 to 0.455 when α value rises from 0.1 to 1.0.

For measuring the chances of occurrence of fuzzy activities, necessity, possibility and credibility measures were used. These measures with pessimistic and optimistic modes and a combination of these two modes, respectively. Thus, the three values of pessimistic, optimistic and a combination of these two modes were determined by solving the fuzzy goal programming according to the credibility, necessity and possibility approaches. The above-mentioned triple efficiency values can lead to better decision-making and sensitivity analyzing by DMs.

Based on the results of fuzzy goal programming models, maximum φ was obtained for lower value of α by the credibility approach.

As shown in Table (6), for some α values, the proposed model does not have a viable solution; this phenomenon occurs after α-cuts. The values obtained for variable coefficients and parameters of various limitations, an infeasible area is supplied. Therefore, there is no solution for the objective function. This is one disadvantages of the fuzzy possibility programming that there is no viable alternative for certain values of α.

Table 6: Summary of the results related to various α values using the possibility measure

	Alpha	φ	Total cost	Maximum completion time	Nodes	CPU time
1	0.1	No feasible solution				
2	0.3	No feasible solution				
3	0.5	No feasible solution				
4	0.7	0.499	7255.912	14.760	3210	15.13
5	0.9	0.533	6333.457	14.437	5974	50.83
6	1	0.541	4689.375	14.362	11268	82.43

Table 7: Summary of the results corresponding to different α values using the necessity measure

	Alpha	φ	Total cost	Maximum completion time	Nodes	CPU time
1	0.1	0.545	6667.677	14.318	6767	61.88
2	0.3	0.549	4239.818	14.285	8550	64.48
3	0.5	0.555	4158.391	14.232	6348	53.78
4	0.7	0.555	4769.915	14.232	7839	55.38
5	0.9	0.581	5558.892	13.982	5391	39.19
6	1	0.586	3856.845	13.930	2773	20.64

Table 8 Summary of the results corresponding to different α values using the credibility measure

	Alpha	φ	Total cost	Maximum completion time	Nodes	CPU time
1	0.1	0.558	5595.900	14.200	2985	19.38
2	0.3	0.554	5615.002	14.237	7261	68.13
3	0.5	0.541	5648.377	14.364	4871	57.67
4	0.7	0.512	5719.925	14.636	6442	39.94
5	0.9	0.474	5813.919	14.993	4316	26.39
6	1	0.455	5862.870	15.179	6676	30.27

8. Conclusion

In this study, we present a bi-objective MILP formulation for simultaneous scheduling and lot-sizing of perishable products under uncertainty. The optimization method incorporates decision making across production schedules, including the quantity of product that can be produced in each duration over the scheduling horizon and the levels of inventory at the end of the day. The objectives of the suggested model are to minimize the overall costs and completion time and to solve the bi-objective model under uncertain demand using a possibilistic fuzzy goal programming method.

The uncertain demand was addressed as fuzzy numbers and we used fuzzy goal programming for solving lot-sizing and scheduling model. The results such as production schedule, inventory level, maximum completion time, and total cost of product at a confidence level of possibility, necessity, and credibility were reported. Accordingly, increasing α-values when applying the possibility and necessity measure causes increase in the φ value rises, but when applying credibility measure, the φ value decreases, when α-values increases, because of the minimization of the objective functions. In our model, necessity and possibility were implemented and evaluated, reflecting the decision makers' pessimistic and optimistic attitudes, as well as the credibility that is the trade-off between necessity and possibility. Better decision-making and sensitivity analysis for DMs can be made based on the three obtained efficiency values. As future work, we will apply robust possibilistic programming (RPP) to overcome the weaknesses of the suggested method and obtain feasible solutions for all α-values.

Another future study may be directed at solving hybrid uncertainties such as facing fuzziness and roughness at the same time.

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