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منحصربه‌فرد بودن اوزان در کارآیی متقاطع با استفاده از مسئله آشفتگی (نمونه موردی: صنایع گاز ایران)

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چکیده:

امروزه تحلیل پوششی داده‌ها به‌صورت گسترده‌ای مورد استفاده قرار گرفته و به‌عنوان یک ابزار غیرپارامتریک کاربردهای فراوانی در ارزیابی و تخمین عملکرد در زمینه‌های متفاوت دارد. ارزیابی مبتنی بر مدل‌های مضربی، گاهی منجر به چندگانی اوزان در بهینگی می‌شود. از سویی وجود وزن صفر در ارزیابی‌ها، یکی از معایب این تکنیک کارآمد محسوب می‌شود. در جهت رفع این مشکل، در این مقاله، روشی مبتنی بر آشفتگی پیشنهاد شده است که نه تنها مشکل چندگانی اوزان در بهینگی را مرتفع می‌کند بلکه مشکل عدم تجانس اوزان را نیز مرتفع می‌سازد. با به‌کارگیری اوزان منحصربه‌فرد در ارزیابی واحدها در کارآیی متقاطع، رتبه منحصربه‌فردی نیز برای واحدها ایجاد می‌گردد. برای تأکید بر قوت روش پیشنهادی مدل معرفی شده بر روی یک مثال واقعی از صنایع گاز کشور ایران پیاده‌سازی می‌شود و با نتایج حاصل از مدل‌های استاندارد، مقایسه می‌گردد. نتایج حاکی از عملکرد بهتر مدل پیشنهادی است.

کلیدواژه‌ها: کارایی متقاطع، تجزیه و تحلیل پوششی داده ها (DEA)، آشوب، وزن منحصر به فرد، عدم تجانس.

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Perturbation and uniqueness of optimal weights in cross-efficiency evaluation: An application to Iranian Gas Companies

Abstract

Data envelopment analysis(DEA) has been extended to cross-efficiency evaluation to better discrimination and ranking of decision making units(DMUs). Unfortunately, the optimal weights generated may not be unique which reduce the usefulness of this powerful method. In addition, due to alternative optimal solutions, zero weights can be seen in cross-efficiency evaluation. To overcom these problems, first the concept of perturbation and generation of unique solution is introduced. Then an alternative evaluation approach containing a perturbed model, is proposed based on the performance analysis without slacks. This modified model can generate unique and non-zero optimal weights, simulteneously. Furthermore, the structure of the model can ensure the dissimilarity of the generated optimal weights. All these factors make the cross-efficiency evaluation results more satisfied and acceptable by all the DMUs. Finally, the proposed approach is applied on a real case study of Iranian Gas Company and the results show that in contrast to standard DEA model the propsed model is more efficient.

Keywords: *Cross–efficiency, Data Envelopment Analysis (DEA), Perturbation, unique weight, dissimilarity.*

1. Introduction

Data envelopment analysis (DEA) is a methodology for measuring the relative efficiency of a set of homogeneous decision making units (DMUs) that use multiple incommensurate inputs to produce multiple incommensurate outputs. In the classical DEA models, the efficiency of a *DMU* is obtained by maximizing ratio of the weighted sum of its outputs to the weighted sum of its inputs, subject to the condition that this ratio does not exceed one for any *DMU*. Traditional DEA models evaluate the DMUs with total weight flexibility, which may cause the situation that many DMUs are evaluated as DEA efficient, and the DEA efficient DMUs cannot be further discriminated [1-5]. Ranking these efficient units seems an interesting subject in DEA literature it has also been discussed in management field [6-8]. Some scholars have also extended the traditional DEA model and proposed modern techniques to improve the discriminative power of DEA. The first group of ranking method includes the super-efficiency evaluation method first proposed by [9] then extended by different researches such as [10] and so on. Super efficiency models suffer from infeasibility and instability. Another improved discrimination method in DEA is common weights which was introduced by [11]. Unlike the traditional DEA method in which each DMU uses its own most favorable weights for efficiency evaluation, the DEA common-weight evaluation method uses a set of common weights for efficiency evaluation of all the DMUs. Further developments of DEA common-weight evaluation can be seen in [12-13]. Among all the methods for increasing the discriminative power of the traditional DEA model, the most commonly used is DEA cross-efficiency evaluation method. DEA cross-efficiency evaluation was originally presented by [14]. Unlike the traditional DEA method which uses a self-evaluated mode with total weight flexibility, the evaluation method uses a peer-evaluated mode [15-16]. The Cross- efficiency method is conduct in two steps. In the first step, the optimal weights of the unit under evaluation with traditional DEA model are obtained. In the second step, by applying the optimal weights of aforementioned step other DMUs will be evaluated. Then, Cross - efficiencies is obtained by the optimal weights selected by all DMUs in the second step. Finally, the average of the Cross- efficiencies of each DMU is called Cross- efficiency score to represent its efficiency performance. Although DEA cross-efficiency evaluation has been widely applied, it still has some shortfalls. The main problem of this method is the problem of non-uniqueness of optimal weights. For each DMU, the optimal weights selected by a traditional DEA model may not be unique, which in turn will cause the problem that different selections of optimal weights will generate different cross-efficiency scores for DMUs. In addition to using different attiutides in DEA litrerature to address the non-uniquness of optimal weights problem in cross-efficiency evaluation, there are some other methos[14,15]. For more details on the drawback of non-uniqueness problems refer to [16-18] .This paper is to address again the non-uniqueness of optimal weights problem in DEA cross-efficiency evaluation. First, the concept of perturbation and uniqueness of optimal weights is discribed. Then to generate strictly positive and dissimilar weights, an alternative model is proposed which contains a perturbed objective function based on the performance analysis without slacks. Further, the main feature of the proposed model is providing weight restrictions and guarantee the uniqueness of the optimal weights of each unit. The proposed approach can perform the cross-efficiency evaluation results more satisfied and acceptable by all the DMUs. Finally, the proposed procedure is employed on evaluation and ranking gas companies in Iran. The rest of the paper is unfolded as follows. Section 2 provides a review to standard DEA model and performane analysis without slacks. The third section introduces the concept of perturburtion and the modified model. Section 4 illustrates the applicability of the proposed model by a real example including Iranian Gas Company. Conclusion will end the paper.

2. Preliminaries

2.1. CCR model (1978)

To describe the DEA efficiency measurement, assume there are n DMUs, $DMU_j\,(j=1,\ldots,n)$ and the performance of each DMU is characterized by a production process of m inputs $x_{ij}\,(i=1,\ldots,m)$ to generate s outputs $y_{rj}\,(r=1,\ldots,s)$. The ratio DEA model also known as CCR[1] model, measures the efficiency of DMU_o by maximizing the ratio of its weighted sum of outputs to its weighted sum of inputs $\theta_o = \dfrac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$, where the maximum is sought subject to the conditions that this ratio does not exceed one for any DMU_j and all the input and output weights are positive. Hence the following fractional model should be solved to obtain the efficiency score of DMU_o:

$$
\begin{aligned}
& Max \quad \theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\
& s.t. \\
& \qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad (1) \\
& \qquad u_r, v_i \ge \varepsilon \quad for \quad all \quad r, i
\end{aligned}
$$

where $\varepsilon > 0$ is a non-Archimedean constraint. This linear fractional programming problem can be reduced to a linear programming model (2) by using Charnes and Copper transformation[19]:

$$
\begin{aligned}
& Max \quad \theta_o = \sum_{r=1}^{s} \bar{u}_r y_{ro} \\
& s.t. \\
& \sum_{i=1}^{m} \bar{v}_i x_{io} = 1 \\
& \sum_{r=1}^{s} \bar{u}_r y_{rj} - \sum_{i=1}^{m} \bar{v}_i x_{ij} \le 0 \qquad (2) \\
& \qquad u_r, v_i \ge \varepsilon \quad for \quad all \quad r, i
\end{aligned}
$$

This model is a constant return to scale (CRS) program and it assumes that the status of all input/output variables are known prior to solving the model. The efficiency ratio θ_o ranges between zero and one, with DMU_o being considered relatively efficient if it receives a score of one. From a managerial perspective, this model delivers assessments and targets with an output maximization orientation.

2.2. Performance Analysis without Slacks

Assume there are n DMUs, $DMU_j\,(j = 1,...,n)$, each one utilizes m inputs $x_{ij}\,(i = 1,...,m)$ to generate s outputs $y_{rj}\,(r = 1,...,s)$ .The following model is proposed by Pourhabib et al. [18] chooses optimal weights for an evaluated unit. Note that the model is equipped with data set which necessarily is scaled up or down.

$$Min \quad \frac{s_o}{\varphi} \qquad (2)$$

$$s.t. \quad \sum_{i=1}^{m} v_{io} x_{io} = 1, \qquad (2-1)$$

$$\sum_{r=1}^{s} u_{ro} y_{ro} + s_o = 1, \qquad (2-2)$$

$$-\sum_{i=1}^{m} v_{io} x_{ij} + \sum_{r=1}^{s} u_{ro} y_{rj} \le 0, \quad j = 1, \cdots, n, j \ne o, \qquad (2-3)$$

$$\varphi \le v_{io} \le 1, \quad i = 1, \cdots, m, \qquad (2-4)$$

$$\varphi \le u_{ro} \le 1, \quad r = 1, \cdots, s, \qquad (2-5)$$

$$u_{ro}, v_{io}, \varphi \ge 0, \quad for\ all\ i\ and\ r \qquad (2-6)$$

As it can be seen in the above model, slack variable s_o is denoted as the deviation variable for DMU_o and φ shows the lower bound for input and output weights. The slack variable s_o shows inefficiency score of DMU_o. In the objective function, the optimal input / output weights are obtained for minimizing s_o and maximizing φ simultaneously. According to the concept of slack variable in DEA literature, the proposed objective function via decreasing s_o tries to minimize the deviation of DMU_o and at the same time, by increasing φ attempts to search a positive lower bound for input / output weights among all feasible multipliers. The important point is model (2) does not require any information about the unit under evaluation. Model (2) is a linear fractional programming problem and it is easy to use Charnes and Cooper [19] transformation to convert it into the following linear equivalent form:

$$
\begin{aligned}
& Min \quad \bar{s}_o \\
& s.t. \quad \sum_{i=1}^{m} \bar{v}_{io} x_{io} = t, \\
& \qquad \sum_{r=1}^{s} \bar{u}_{ro} y_{ro} + \bar{s}_o = t, \\
& \quad -\sum_{i=1}^{m} \bar{v}_{io} x_{ij} + \sum_{r=1}^{s} \bar{u}_{ro} y_{rj} \le 0, \quad j = 1, \cdots, n, \ j \ne o, \qquad (3) \\
& \qquad 1 \le \bar{v}_{io} x_{io} \le t, \quad i = 1, \cdots, m, \\
& \qquad 1 \le \bar{u}_{ro} y_{ro} \le t, \quad r = 1, \cdots, s, \\
& \qquad \bar{u}_{ro}, \bar{v}_{io}, \bar{s}_o \ge 0, \quad for\ all\ i\ and\ r
\end{aligned}
$$

In which $\frac{1}{\varphi} = t,\ \bar{s}_o = t\, s_o,\ \bar{v}_i = t\, v_o,\ \bar{u}_r = t\, u_o.$ obviously, model(2) is feasible and bounded. It is proved that DMU_o is efficient if $s_o^* = 0$ in any optimal solution of model (2). Furthermore, model (2) is feasible and in optimality $\varphi^* > 0$. Now the question arises whether model (2) or the equivalent linear form, model (3), has unique optimal solutions? The answer is obviously negative. The above proposed approach has possible existence of alternative optimal for the input/output weights, which may lead to different cross-efficiency scores and different ranking. In the next section, this approach is modified so that not only gurantees non-zero weights and avoids unrealistic weights but also produces unique optimal solution.

3. Proposed Approach

In this section, we first examine model (2) in order to obtain a set of unique optimal weights. Then based on this unique optimal set and applying cross-efficiency method, units are ranked. As mentioned before, model (2) has alternative optimal solutions which can produce different Cross-efficiency scores lead to different ranking. In order to obtain the unique and optimal set of weights, model (2) is modified in order to produce unique Cross-efficiency scores. The alternative approach guarantees non-zero weights or equivalent zero slacks in DEA assessments. Simultaneously, avoids weights dissimilarity in term of value of multipliers. In addition to above mentioned advantages, this unique optimal solution employs for cross-efficiency evaluation. In order to shed a light on the idea behind the modified model, the relation between the primer and dual model is under our lens. Suppose that the primal linear program is given in the following form:

$$\begin{aligned} Min \quad & z(x) = cx \\ s.t. \quad & Ax = b \qquad (4) \\ & x \geq 0 \end{aligned}$$

As far as we know, the dual form of model (4) is as follows:

$$\begin{aligned} Max \quad & wb \\ s.t. \quad & wA \leq c \qquad (5) \\ & w \quad unrestricted \end{aligned}$$

To best of our knowledge, there is exactly one dual variable for each primal constraint and exactly one dual constraint for each primal variable. The variable matrix A, in model(4) is a matrix of order $m*n$ with rank m. Notably, there are at most $\sum_{r=1}^{m-1} \binom{n}{r}$ subsets of column vectors of A with $m-1$ or less column vectors in them. Actually, the linear hull of these subsets is a subspace of R^m with dimension $m-1$ or less. The Linear Programming (4) is degenerate if and only if the right hand side vector(b) stays in one of these subspaces. The set of all $b \in R^m$ that make model (4) degenerate is the set of all right hand sides (b) stay in a finite number of subspaces of dimension $m-1$ or less. Statistically speaking, almost all right hand sides will make model (4) nondegenerate. Hence, if model (4) is degenerated, the right hand side vector (b) can be perturbed to a point in its neighborhood that is not belonging to any of the finite number of subspaces discussed earlier; thus model (4) is nondegenerate. Such perturbation that makes model(4) nondegenerate is replacing the vector b by $b(\varepsilon) = b + (\varepsilon, \varepsilon^2, ..., \varepsilon^m)^t$ where ε is an non Archimedean arbitrarily small positive number and $\varepsilon^r = \varepsilon \times \varepsilon \times ... \times \varepsilon$ implies that ε multiplies r times. According to the following theorem, to produce unique optimal solutions, an Archimedean infinitesimal number ε should be added to the right hand side of the model(4). As a result, the new perturbed problem is nondegenerate. Hence, optimal solutions in dual format is unique if primal problem is nondegenerate.

Theorem 1: Given $b \in R^m$, there exists a positive number $\varepsilon_1 > 0$, such that whenever $0 < \varepsilon < \varepsilon_1$, the below perturbed problem is nondegenerate.

$$
\begin{aligned}
& Min \quad z(x) = cx \\
& s.t. \quad Ax = b(\varepsilon) \\
& \qquad\quad x \geq 0 \\
& that \;\; b(\varepsilon) = b + (\varepsilon, \varepsilon^2, ..., \varepsilon^m)^t
\end{aligned}
$$

Proof: refer to Murty [20] (Chapter 10).

Theorem 2: If an optimal basic feasible solution in primal problem is nondegenerate then dual optimal solution is unique.

Proof: refer to Bazara [21] (Chapter 6).

Equipped with this fact, the primal linear programming model (3) is perturbed via adding up the infinitesimal number epsilon to the right hand side. Perturbation leads to following model:

$$
\begin{aligned}
& Max \quad m+n \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + \theta x_{io} + m - r \leq 0 + \varepsilon^i, i = 1, 2, \ldots, m \qquad (6) \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} + \eta y_{ro} + n - z \leq 0 + \varepsilon^{m+r}, r = 1, 2, \ldots, s \\
& \qquad \eta \leq 1 + \varepsilon^{m+s+1}, \\
& \qquad -\theta - \eta + r + z \leq 0 + \varepsilon^{(m+s+2)}, \\
& \qquad \lambda_j, m, n, r, z \geq 0, \qquad \theta, \eta \; free
\end{aligned}
$$

The dual format of model (6) is the following program:

$$
\begin{aligned}
& Min \quad (\sum_{i=1}^{m} v_i \varepsilon^i + \sum_{r=1}^{s} u_r \varepsilon^{m+r} + s_o (1 + \varepsilon^{m+s+1}) + \varepsilon^{m+s+2}) / \varphi \\
& s.t. \quad \sum_{i=1}^{m} v_{io} x_{io} = 1, \\
& \qquad \sum_{r=1}^{s} u_{ro} y_{ro} + s_o = 1, \\
& \qquad -\sum_{i=1}^{m} v_{io} x_{ij} + \sum_{r=1}^{s} u_{ro} y_{rj} \leq 0, \quad j = 1, \cdots, n, j \neq o, \qquad (7) \\
& \qquad \varphi \leq v_{io} \leq 1, \quad i = 1, \cdots, m, \\
& \qquad \varphi \leq u_{ro} \leq 1, \quad r = 1, \cdots, s, \\
& \qquad u_{ro}, v_{io}, \varphi \geq 0, \quad for \; all \; i \; and \; r
\end{aligned}
$$

The above model (7) has all properties of model (2). It guarantees the positivity of the weights and lead to non-ignorance of no input and output variables. Simultaneously, the model avoids weights dissimilarity in the values of multipliers. Employing the above model in efficiency evaluation generates a unique cross-efficiency matrix. So, the Cross-efficiency scores and ranking will be unique. However, based on the proposed approach in alternative optimal solutions, DMU_o selects a unique set of optimal weights to be used in Cross-efficiency evaluation. Based on obtained unique optimal weight sets, we can now calculate cross -efficiency scores. If possible optimal solution of model (7) can be defined as $(v_{1o},\dots.v_{mo},u_{1o},\dots,u_{ro})$ then the cross-efficiency score for DMU_o, applying the unique profile of weights is obtained as: $E_{jo}=\frac{\sum_{r=1}^{s}u_{ro}y_{rj}}{\sum_{i=1}^{m}v_{io}x_{ij}} \quad ,j=1,\dots,n \qquad (8)$. The cross- efficiency score of DMU_j is then defined as the average of E_{jo} as $E_j^{-}=\frac{1}{n}\sum_{j=1}^{n}E_{jo}$. This score is used as an alternative efficiency score for DMU_j. Note that the unique optimal weights obtained from model (7) lead to a unique cross- efficiency score and ranking.

4. Numerical Example

4.1. Simple example

This section illustrates the proposed model in assessing a small-scale example which consists of five DMUs. The DMUs fed up with two inputs to produce one fixed output. Table 1 depicts the data set and the CCR efficiency for this example.

Table1: The data for simple example

DMU	X_1	X_2	Y	EFF^{CCR}
A	1	10	1	1
B	2	5	1	1
C	4	2	1	1
D	6	1	1	1
E	12	0.5	1	1

We apply model (7) to data set of Table (1). The results are reported in Table (2). As the Table shows, input and output weights are drawn in columns two till four. Also, efficiency scores are demonstrated in the last column.

Table2: the results of proposed approach

DMU	V_1^*	V_2^*	U^*	EFF^{new}
A	0.333	0.066	1	1
B	0.187	0.125	1	1
C	0.166	0.166	1	1
D	0.125	0.250	1	1
E	0.055	0.666	1	1

Notably, GAMS software on a machine with CPU: Intel Pentium 4 at 2 GHz, RAM: 512 MB is used to our calculation. As it is recorded, optimal weights for inputs and outputs represent the coefficient of hyperplanes that touch the production set. More concretely, we have the following hyperplanes:

$$H_A = \{ (x, y) : y - 0.333x_1 - 0.066x_2 = 0 \} \cap T_c$$
$$H_B = \{ (x, y) : y - 0.187x_1 - 0.125x_2 = 0 \} \cap T_c$$
$$H_C = \{ (x, y) : y - 0.166x_1 - 0.166x_2 = 0 \} \cap T_c$$
$$H_D = \{ (x, y) : y - 0.125x_1 - 0.250x_2 = 0 \} \cap T_c$$
$$H_E = \{ (x, y) : y - 0.055x_1 - 0.666x_2 = 0 \} \cap T_c$$

Figure (1) shows the production set in two-dimensional space along with the hyperplanes obtained from model (7).

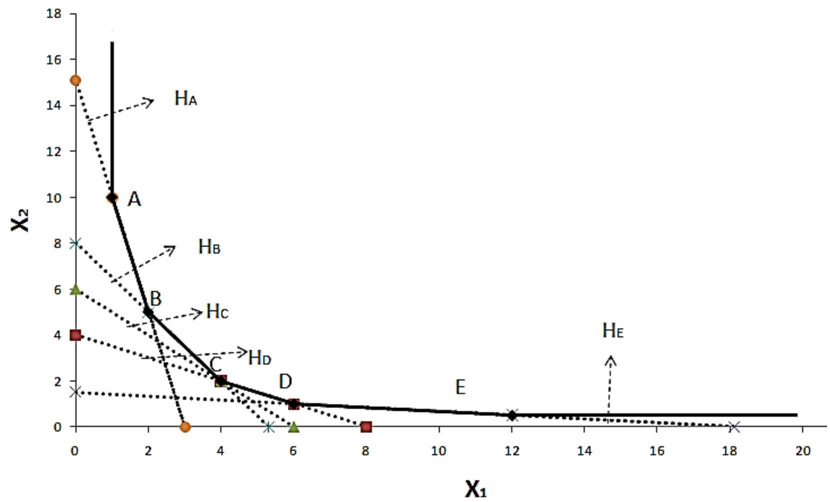

Figure 1: The production set for simple example

Now Applying cross- efficiency method on simple data set of Table (1), the matrix of cross -efficiency related to proposed approach is presented in Table 3.

Table3: Matrix of cross-efficiency

DMU	1	2	3	4	5	scores	rank
1	1	0.6957	0.546	0.381	0.149	0.55421	5
2	1	1	0.857	0.667	0.29	0.7629	3
3	0.6819	1	1	1	0.643	0.86507	1
4	0.4839	0.8	0.857	1	1	0.82836	2
5	0.248	0.4324	0.48	0.615	1	0.55519	4

As Table (3) shows the unique set of weights are applied in cross-efficiency evaluation. What's more, applying relation (8) lead to a unique rank.

4.2. Applicable example

National Iranian Gas Company (NIGC), was founded in 1965 and is responsible for different activities of extraction, transmission and distribution of gas in Iran. The gas refinery capacity rate of NIGC is 446 million cubic meters per day. With 28,000 km of high pressure transmission and 150,000 km of distribution lines, NIGC provides the consumption of sectors domestic/business (44%), power plants (30%) and industrial (26%). The 744 cities and 7344 villages are covered by NIGC. We apply our model to the data set used by [22]. Two factors are selected as inputs and three factors are considered as outputs. Table (4) reports the input and output measures.

Table 4: The inputs and outputs for Gas Company

	input	Output
1	Kilometer of network	Delivered volumes (million cubic m)
2	Number of employees	Number of customer (*1000)
3		Actual/plan performance (%)

The data set consists of 27 branches of gas company in IRAN as shown in Table (5).

Table 5: The data for 27 Iranian provincial gas company in 2008

DMU	Company name	X_1	X_2	Y_1	Y_2	Y_3
1	Ardabil	3167.7	148	976.3	209889	121.1
2	Azarbayjangharbi	5177.8	197	2820.3	420070	94.3
3	Azarbayjansharghi	10664.8	355	6645.4	831751	136.1
4	Bushehr	978.8	41	5477.2	7911	91.5
5	Chaharmahal-o-bakhtiari	3411.5	150	978.1	165384	94.5
6	Esfahan	16545.2	754	14816	1069452	222.5
7	Fars	11088.9	512	7913	630757	134.2
8	Ghazvin	2614.9	141	3874.1	195170	116.7
9	Ghom	2242.2	123	1821.9	227171	130.5
10	Gilan	9398.4	384	4529	465329	102.1
11	Golestan	4654.5	186	1222.2	267941	147.2
12	Hamedan	5424.7	281	2526.2	317115	139.4
13	Ilam	497.6	80	29.6	11345	65.3
14	Kerman	6171.4	214	7119	269039	97.7
15	Kermanshah	3515.1	143	2315	273419	59.2
16	Khorasan razavi	11111.8	448	8233.7	1118628	182.8

17	Khorasanshomali	2149.9	88	2269.2	120477	84.6
18	Khozestan	7508.7	543	11366.9	434583	72.7
19	Kohkiloyeh-o-boirahmad	1791.4	117	437.7	72779	51.9
20	Kurdestan	2916.5	127	1732.2	193985	129.9
21	Lorestan	2645.2	156	1328.6	240223	110.9
22	Markazi	4614.2	204	3970	290994	192.6
23	Mazandaran	11445.1	783	5842	670312	102.1
24	Semnan	2784.4	139	1054	145934	114.4
25	Tehran	20981.1	1130	24353.4	2091476	105.3
26	Yazd	3993.5	142	2176.4	206249	108
27	Zanjan	1992.7	117	888	138526	67.9

The CCR efficiency scores and the optimal weights of inputs and outputs are given in Table 6.

Table 6: The optimal weights and CCR efficiency

DMU	CCR efficiency	v_1	v_2	u_1	u_2	u_3
1	0.6578	0.000305	0.0002278	0	0.0000031	0.0000311
2	0.8545	0	0.0050761	0	0.000002	0.0000211
3	0.9486	0	0.0028169	0.0000197	0.000001	0
4	1	0.0010217	0	0.0001826	0	0
5	0.4822	0.0002838	0.0002119	0	0.0000029	0.0000289
6	0.6957	0.0000482	0.0002689	0.0000099	0.0000005	0
7	0.5984	0.0000717	0.0004002	0.0000147	0.0000008	0
8	0.851	0.0003824	0	0.0000639	0.0000031	0
9	1	0.000446	0	0.0000242	0.0000042	0.0000298
10	0.5102	0.0000866	0.0004836	0.0000178	0.0000009	0
11	0.5792	0	0.0053763	0	0.0000021	0.0000223
12	0.578	0.0001783	0.0001171	0.0000079	0.0000018	0
13	0.2367	0.0020096	0	0	0.0000197	0.0001989
14	0.6711	0	0.0046729	0.0000326	0.0000016	0

15	0.7858	0.0002318	0.0012939	0.0000475	0.0000025	0
16	1	0.0000735	0.00041	0.0000151	0.0000008	0
17	0.6624	0.0003786	0.0021132	0.0000777	0.000004	0
18	0.7209	0.0001332	0	0.0000222	0.0000011	0
19	0.4015	0.0005582	0	0	0.0000055	0.0000553
20	0.6683	0.0002758	0.0015394	0.0000566	0.0000029	0
21	0.8963	0.000378	0	0	0.0000037	0
22	0.6807	0.0001738	0.0009701	0.0000357	0.0000019	0
23	0.5799	0.0000874	0	0.0000039	0.0000008	0
24	0.5202	0.0003462	0.0002585	0	0.0000035	0.0000353
25	1	0.0000366	0.0002045	0.0000075	0.0000004	0
26	0.6141	0	0.0070423	0.0000492	0.0000025	0
27	0.6861	0.0005018	0	0	0.000005	0

Table 6 shows four units #4, 9, 16 and 25 are efficient. As the last five columns of Table6 records, some of the input/output weights are zero. Therefore, the variables' dependency on these weights do not participate in efficiency calculation Now, the proposed approach, model (7), is applied on this data set. The new efficiency scores, Eff_d^{new} , and the optimal weights are listed in Table 7.

Table 7. New efficiency scores and optimal weights for Iranian gas company

DMU	slack	V_1	v_2	v_3	u_1	u_2	Eff_d^{new}
1	0.344199	0.00030463	0.00023666	0.00000309	0.00000309	0.00002927	0.655801
2	0.146558	0.00000201	0.00502322	0.00000201	0.00000201	0.00001988	0.853442
3	0.054006	0.00000099	0.00278705	0.00001795	0.00000099	0.00000099	0.945994
4	0	0.00000968	0.02415904	0.00000968	0.00000968	0.00009559	1
5	0.518616	0.00028344	0.00022021	0.00000288	0.00000288	0.00002723	0.481384
6	0.338721	0.00005804	0.00005275	0.00000316	0.00000057	0.00000408	0.661279
7	0.43374	0.00008804	0.00004628	0.00000089	0.00000089	0.00000089	0.56626

8	0.157177	0.00038225	0.00000315	0.00005853	0.00000315	0.00000315	0.842823
9	0	0.00042776	0.00033233	0.00000434	0.00000434	0.00004109	1
10	0.507133	0.00010416	0.00005476	0.00000105	0.00000105	0.00000105	0.492867
11	0.422549	0.00000213	0.00532295	0.00000213	0.00000213	0.00002106	0.577451
12	0.422223	0.00017946	0.00009434	0.00000181	0.00000181	0.00000181	0.577777
13	0.765404	0.0020065	0.00001955	0.00001955	0.00001955	0.00018704	0.234596
14	0.34403	0.00000165	0.00462534	0.00002979	0.00000165	0.00000165	0.65597
15	0.226593	0.00027853	0.00014642	0.0000028	0.0000028	0.0000028	0.773407
16	0	0.00000089	0.00221014	0.00000089	0.00000089	0.00000089	1
17	0.346671	0.0003877	0.00189186	0.00007075	0.00000409	0.00000409	0.653329
18	0.290889	0.0001331	0.0000011	0.00002038	0.0000011	0.0000011	0.709111
19	0.599792	0.00055787	0.00000546	0.00000546	0.00000546	0.00000546	0.400208
20	0.336748	0.00033166	0.00025766	0.00000337	0.00000337	0.00003186	0.663252
21	0.105999	0.00037783	0.0000037	0.0000037	0.0000037	0.0000037	0.894001
22	0.352814	0.00020835	0.00018936	0.00001135	0.00000206	0.00001463	0.647186
23	0.421852	0.00008732	0.00000085	0.00000085	0.00000085	0.00000085	0.578148
24	0.48169	0.00034996	0.00018397	0.00000352	0.00000352	0.00000352	0.51831
25	0	0.00003774	0.00018417	0.00000689	0.0000004	0.0000004	1
26	0.415185	0.00000279	0.00696374	0.00000279	0.00000279	0.00002755	0.584815
27	0.315041	0.00050154	0.00000491	0.00000491	0.00000491	0.00000491	0.684959

The results of Table 6 and 7 show that our proposed efficiency scores are less than or equal to the scores obtained from CCR model. The mean efficiency scores in our approach and CCR model are 0.69082, 0.69924, respectively. Note that no input and output variables cannot be ignored in our proposed approach, because the proposed approach guarantees the positivity of the weights while the zero weights occur in the CCR model. Now we compare the cross- efficiency evaluation method using the weights obtained from two different approaches. The Cross-efficiency scores and ranking of efficient unit for CCR and Our proposed approach are listed in Table 8.

Table 8: The results of Cross-efficiency evaluation

DMU	CCR Cross scores	Our proposed approach Cross scores
1	0.547819	0.621981
2	0.725112	0.812273
3	0.734574	0.817581
4	0.876274(2*)	0.684147(4)
5	0.413213	0.465087
6	0.594262	0.635624
7	0.514037	0.554218
8	0.698248	0.722704
9	0.834795(4)	0.927539(3)
10	0.448674	0.492461
11	0.495747	0.563424
12	0.488259	0.542434
13	0.139228	0.168773
14	0.489708	0.489991
15	0.695954	0.770167
16	0.891034(1)	0.993561(1)
17	0.558777	0.581074
18	0.532549	0.537129
19	0.313588	0.354504
20	0.588059	0.648845
21	0.714399	0.808256
22	0.58403	0.625697
23	0.459601	0.510946
24	0.438797	0.489826
25	0.856021(3)	0.929713(2)
26	0.491425	0.53755
27	0.552951	0.62185

*The values in parenthesis show the rank of efficient units

Note that all of inputs and outputs are important for the decision makers and executive managers of companies. It should be pointed out that, all optimal weights achieved through model (7) are positive. As a result, no inputs and outputs are ignored in the cross- efficiency score. In contrast, existence of zero weights in CCR model leads to unrealistic cross-efficiency score. The second and third columns of Table (8) show the Cross-efficiency scores in the

CCR and our proposed approach. According to these scores, DMU number#16 (Khorasan Razavi branch) has the top rank among the other branches in both models.

5. Conclusions and Future Research

Because of its good ability in evaluation and ranking of DMUs, DEA cross-efficiency evaluation has been widely applied in a variety of areas. However, a problem occurs with this evaluation methodology because of non-uniqueness and zero optimal weights. The existence of unrealistic weights in DEA models lead to an incorrect assessment in efficiency analysis. This paper contributes to the continuing debate of non-uniqueness of optimal weights in DEA cross-efficiency evaluation. To be specific, this paper proposed an alternative evaluation in order to solve the problem. Firstly, the concept of perturbation and uniqueness of results were introduced. Then a modified model with a perturbed objective function was proposed based on performance evaluation without slacks. The modified model is a developed form of standard DEA models in its multiplier orientation. Three issues are therewith addressed. First, the concept of perturbation is incorporated in DEA cross-efficiency evaluation. Second, the modified model guarantees unique weight portfolio and avoids disimilarity in DEA assessments. Third, this unique set lead to unique cross-efficiency score and ranking. Application of the original DEA model and the proposed model on a real case example of "Iranian Gas Company" recognize that the modified model can generate strictly positive and dissimilar optimal weight. The advantage of the proposed model can be listed as follows: The proposed efficiency scores are less than or equal to the scores obtained from standard DEA(CCR) model. Since, all of inputs and outputs are important for the decision makers and executive managers of companies, the proposed approach does not ignore input and output variables and guarantees the positivity of the weights while the zero weights occur in the CCR model. It should be pointed out that, all optimal weights achieved through our proposed model are strictly positive. Notably, existence of zero weights in CCR model leads to unrealistic cross-efficiency score. But, the proposed model generates unique optimal weights based on the perturbed problem. Furthermore, the perturbed model generates a unique cross-efficiency matrix. So, the Cross-efficiency scores and ranking are unique. For future work, it is recommended to apply an upper and lower bound for inputs and outputs weight, in this case there is no need to scale all data as we have done in this model.

6. Endnote

1. Charnes, Cooper and Rhodes(CCR)

7. References

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